Mathematics Faculty

Further Maths Transition Booklet

Contents

QUESTIONS

ANSWERS

Compulsory Textbooks: There are 3 compulsory textbooks you must purchase

1) **Pearson Edexcel AS and A Level Further Mathematics- Core Pure Mathematics 1**
   ISBN: 9781292183336
2) **Pearson Edexcel AS and A Level Further Mathematics- Further Mechanics**
   ISBN: 9781292183312
3) **Pearson Edexcel AS and A Level Further Mathematics- Further Statistics**
   ISBN: 9781292183374

NB: The textbook listed above must be purchased by all students on this course by 18/09/20

These are some useful websites:

www.examsolution.co.uk   physicsandmathstutor.com   furthermaths.org.uk/studentarea

www.mymaths.co.uk   www.khanacadamy.org/

Please note that Further Maths students need to complete the exercises in this Booklet only and not AS Booklet.

Edexcel AS Mathematics Surds and indices

Indices

Do not use a calculator in this exercise.

1. Find:

(i) 3^4 (ii) 2^6 (iii) $4^{1/2}$

(iv) 6^0 (v) 5^{-2} (vi) $64^{1/3}$

(vii) $16^{-1/2}$ (viii) $8^{5/3}$ (ix) $36^{-3/2}$

(x) $\left(\frac{1}{2}\right)^{-1}$ (xi) $\left(\frac{25}{9}\right)^{-1/2}$ (xii) $\left(\frac{27}{64}\right)^{-2/3}$

2. Simplify the following:

(i) $3^{11} \times 3^{-4} \div 3^3$ (ii) $\left(2^5\right)^3 \times \left(2^7\right)^{-2}$ (iii) $\frac{5^6}{5^5 \times 5^3}$

3. Simplify:

(i) $2^3 \times 16^{\frac{1}{2}}$ (ii) $\frac{3^5 \times 5^3}{\sqrt{81 \times 25}}$

Surds

Do not use a calculator in this exercise.

1. Write these in terms of the simplest possible surd.

(i) $\sqrt{8}$ (ii) $\sqrt{50}$ (iii) $\sqrt{48}$

(iv) $\sqrt{216}$

2. Simplify the following
   (i) $\left(1+\sqrt{2}\right)+\left(3-2\sqrt{2}\right)$ (ii) $\left(5\sqrt{2}-2\sqrt{3}\right)-\left(\sqrt{2}+3\sqrt{3}\right)$
   (iii) $2\left(\sqrt{5}-3\sqrt{3}\right)+3\left(2\sqrt{5}+\sqrt{3}\right)$

3. Multiply out the brackets and simplify as far as possible.
   (i) $\left(1+\sqrt{2}\right)\left(3-\sqrt{2}\right)$ (ii) $\left(2-\sqrt{3}\right)\left(3+2\sqrt{3}\right)$

4. Rationalise the denominators of the following.
   (i) $\dfrac{3}{\sqrt{3}}$ (ii) $\dfrac{1}{\sqrt{5}}$
   (iii) $\dfrac{1+\sqrt{2}}{\sqrt{2}}$ (iv) $\dfrac{1}{\sqrt{3}+1}$
   (v) $\dfrac{\sqrt{2}}{2-\sqrt{2}}$

Simultaneous Equations

1. Solve the following simultaneous equations:
   (i) $2x+5y=11$, $2x-y=5$ (ii) $x+2y=6$, $4x+3y=4$
   (iii) $3a-2b=4$, $5a+4b=3$ (iv) $2p-5q=5$, $3p-2q=-9$

2. Solve the following simultaneous equations:
   (i) $x-y=-1$, $3x+2y=7$ (ii) $2x+y=0$, $x-3y=7$
   (iii) $y-5x=-8$, $x+3y=0$ (iv) $x=2y-1$, $-x+3y=-1$

The Quadratic Formula

1. Without solving the equation, state how many solutions there are for each of the following quadratic equations:
   (i) $3x^2+2x+5=0$ (ii) $2x^2-3x-2=0$
   (iii) $5x^2-6=0$ (iv) $4x^2-8x+4=0$

2. Use the quadratic formula to solve these equations. Give your answers in exact form.

(i) $x^2 + 4x + 1 = 0$ (ii) $x^2 - 3x - 1 = 0$

(iii) $2x^2 + 2x - 3 = 0$

Inequalities

1. Solve the following linear inequalities.

(i) $2x + 3 < 10$ (ii) $5x + 3 \geq 2x - 9$

(iii) $3x - 1 > 7 - x$ (iv) $4x + 1 \leq 6x - 7$

(v) $5x + 2 > -7$ (vi) $3x - 11 \leq 5 + 4x$

2. (i) Write $x^2 - 11x + 24$ in factorised form.

(ii) Sketch the graph of $y = x^2 - 11x + 24$, labelling the values of x where the graph crosses the x-axis.

(iii) Use your graph to write down the solution of

$$x^2 - 11x + 24 \geq 0$$

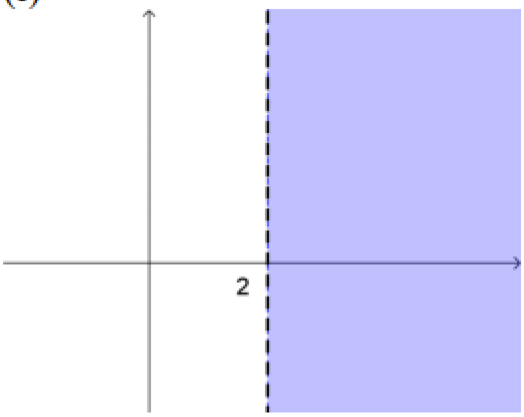
3. Solve the following quadratic inequalities.

(i) $x^2 - 4x - 12 \le 0$ (ii) $x^2 - 7x + 6 > 0$

(iii) $x^2 + 2x - 15 \ge 0$

4. Write down an inequality to describe the shaded area in each of these diagrams.

(i)



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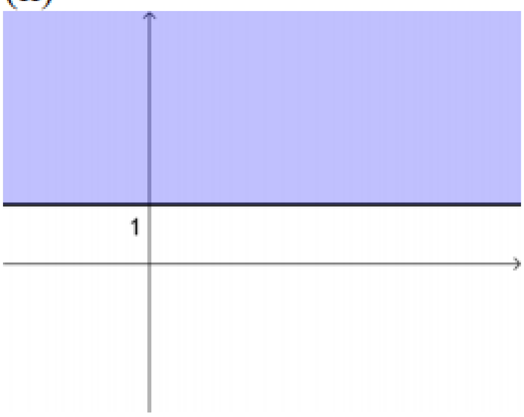

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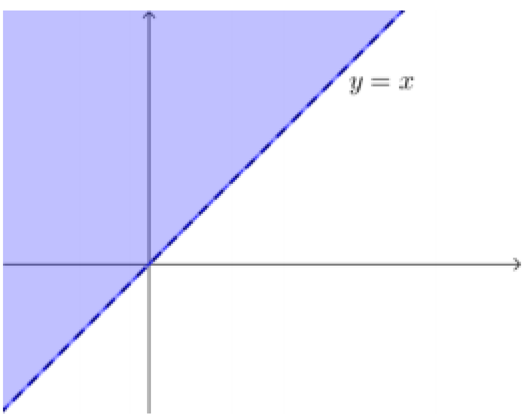

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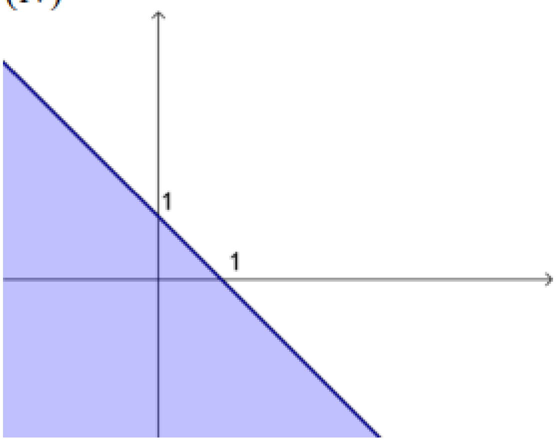


Algebraic Fractions

1. Simplify

(i) $\dfrac{x^2 - 5x}{x - 5}$ (ii) $\dfrac{x^2 + 6x - 16}{(x-2)(x-8)}$

(iii) $\dfrac{x^2 - a^2}{(x-a)^2}$ (iv) $\dfrac{v - 3}{6 - 2v}$

2. Write as a single fraction in its simplest form

(i) $\dfrac{x^2 + 3x - 4}{8} \times \dfrac{2}{3x - 3}$ (ii) $\dfrac{3}{x+2} \times \dfrac{x^2 - 4x - 12}{x^2 - 2x - 24}$

3. Write as a single fraction in its simplest form

(i) $\dfrac{5c+15}{2} \div \dfrac{c^2-9}{4}$ (ii) $\dfrac{x^2-x}{2x+1} \div \dfrac{x^2+2x-3}{2x^2-3x-2}$

4. Express as a single fraction

(i) $\dfrac{x}{2} + \dfrac{3x}{5}$ (ii) $\dfrac{2}{3x} - \dfrac{1}{4x}$

5. Express as a single fraction

(i) $\dfrac{2}{x+3} + \dfrac{1}{x-5}$ (ii) $\dfrac{5}{2x-3} - \dfrac{2}{4x+1}$

6. Express as a single fraction

(i) $\dfrac{1}{2(x+5)} - \dfrac{1}{4(x+1)}$

7. Divide

(i) $3x^3 - x^2 + 2x - 4$ by $x + 2$

(ii) $x^4 + 2x^2 - x - 1$ by $x^2 + 1$

(iii) x^3 by $x^2 + 2$

(iv) $6x - 2$ by $2x + 3$

(v) $3x^2 + 1$ by $x^2 - 2x - 1$

The Sine and Cosine Rules

1. Find two possible values of c in triangle ABC given that $a = 16$ cm, $b = 10$ cm, and B = 30°.

2. Solve the triangle PQR in which $p = 8$ cm, $q = 9$ cm and $r = 10$ cm.

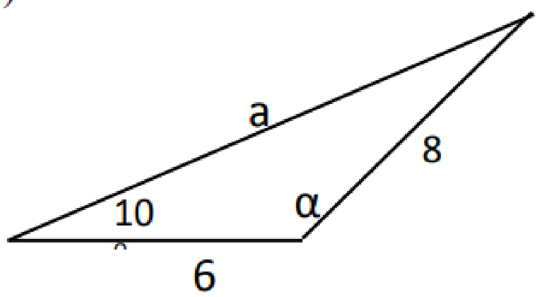
3. In triangle XYZ, X = 100°, Y = 30° and XY = 10 cm. Calculate the area of the triangle.

4. The area of a triangle is 12 cm^2. Two of the sides are of lengths 6 cm and 7 cm. Calculate possible lengths for the third side.

5. A ship S is 6.8 km from a lighthouse on a bearing of 310°. A second ship T is 8.4 km from the lighthouse on a bearing 075°. Calculate ST and the bearing of T from S correct to the nearest degree.

6. Find all the lettered edges and angles in the figures in the following diagrams:

(i)



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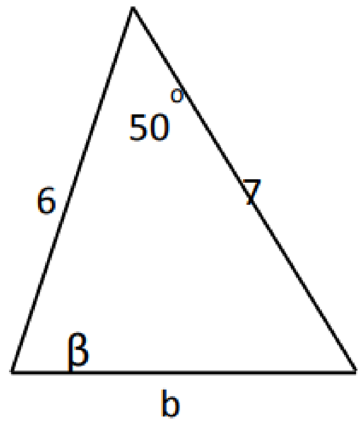


Points and straight lines

1. Given the points A(3, 1), B(6, y) and C(12, -2) find the value(s) of y for which
   (i) the line AB has gradient 2
   (ii) the distance AB is 5

2. Find the equations of the following lines.
   (i) parallel to $y = 4x - 1$ and passing through (2, 3)
   (ii) perpendicular to $y = 2x + 7$ and passing through (1, 2)

3. Find the equation of the line AB in each of the following cases.
   (i) A(1, 6), B(3, 2)
   (ii) A(8, -1), B(-2, 3)

4. The point E is (2, -1), F is (1, 3), G is (3, 5) and H is (4, 1).
   Show, by calculation that EFGH is a parallelogram.
   Is EFGH also a rhombus? Explain your answer.

5. P is the point (2, 1), Q is (6, 9) and R is (10, 2).
   (i) Sketch the triangle PQR.
   (ii) Prove that triangle PQR is isosceles.
   (iii) Work out the area of triangle PQR.

Circles

1. Find, in the form $x^2 + y^2 + px + qy = c$, the equation for each of the following circles.
   (i) centre (0, 0), radius 6
   (ii) centre (3, 1), radius 5

2. For each of these circles, write down the coordinates of the centre and the radius.
   (i) $x^2 + y^2 = 100$
   (ii) $(x-2)^2 + (y-7)^2 = 16$
   (iii) $(x+3)^2 + (y-4)^2 = 4$
   (iv) $(x+4)^2 + (y+5)^2 = 20$

3. For each of these circles, find the coordinates of the centre and the radius.
   (i) $x^2 + y^2 + 4x - 5 = 0$
   (ii) $x^2 + y^2 - 6x + 10y + 20 = 0$

4. The point C is (4, -2) and the point A is (6, 3).
   Find the equation of the circle centre C and radius CA.

5. The points A (2, 0) and B (6, 4) form the diameter of a circle. Find the equation of the circle.

Coordinate Geometry

1. A line l_1 has equation $5y + 4x = 3$.
   (i) Find the gradient of the line. [1]
   (ii) Find the equation of the line l_2 which is parallel to l_1 and passes through the point (1, -2). [3]

2. Describe fully the curve whose equation is $x^2 + y^2 = 4$. [2]

3. The coordinates of two points are A (-1, -3) and B (5, 7). Calculate the equation of the perpendicular bisector of AB. [4]

4. Show that the line $y = 3x - 10$ is a tangent to the circle $x^2 + y^2 = 10$. [4]

5. The line $y = 2x - 3$ meets the x-axis at the point P, and the line $3y + 4x = 8$ meets the x-axis at the point Q. The two lines intersect at the point R.
   (i) Find the coordinates of R. [4]
   (ii) Find the area of triangle PQR. [3]

Polynomial Functions and Graphs

1. Given that $f(x) = x^3 + 2x^2 - 5x + 4$ and $g(x) = x^3 - 3x^2 + 1$, find
   (i) $f(x) + g(x)$
   (ii) $f(x) - g(x)$

2. Given that $p(x) = 2x^3 - 5x^2 + 3x - 2$ and $q(x) = x^3 - 2x^2 + 1$, find
   (i) $q(x) - p(x)$
   (ii) $2p(x) + 3q(x)$

3. Expand the brackets and simplify the following as far as possible:
   (i) $(x-2)(2x^2 - 3x + 1)$
   (ii) $(3x-2)(x^3 - 2x + 4)$

4. Sketch the following graphs:
   (i) $y = (x+1)(x-3)(x+4)$
   (ii) $y = (x+2)^2(2x-1)$

Indices Answers

1. (i) $3^4 = 3\times3\times3\times3 = 81$

   (ii) $2^6 = 2\times2\times2\times2\times2\times2 = 64$

   (iii) $4^{1/2} = \sqrt{4} = 2$

   (iv) $6^0 = 1$

   (v) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

   (vi) $64^{1/3} = \sqrt[3]{64} = 4$

   (vii) $16^{-1/2} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

   (viii) $8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

   (ix) $36^{-3/2} = \frac{1}{\left(\sqrt{36}\right)^3} = \frac{1}{6^3} = \frac{1}{216}$

   (x) $\left(\frac{1}{2}\right)^{-1} = \left(2^{-1}\right)^{-1} = 2^1 = 2$

   (xi) $\left(\frac{25}{9}\right)^{-1/2} = \left(\frac{9}{25}\right)^{1/2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

   (xii) $\left(\frac{27}{64}\right)^{-2/3} = \left(\frac{64}{27}\right)^{2/3} = \left(\sqrt[3]{\frac{64}{27}}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

2. (i) $3^{11}\times3^{-4}\div3^3 = 3^{11-4-3} = 3^4 = 81$

   (ii) $\left(2^5\right)^3\times\left(2^7\right)^{-2} = 2^{15}\times2^{-14} = 2^{15-14} = 2^1 = 2$

   (iii) $\frac{5^6}{5^5\times5^3} = 5^{6-5-3} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

3. (i) $2^3\times16^{\frac{1}{2}} = 2^3\times\left(2^4\right)^{\frac{1}{2}}$

   $= 2^3\times2^2$

   $= 2^5 \ (= 32)$

   (ii) $\frac{3^5\times5^3}{\sqrt{81\times25}} = \frac{3^5\times5^3}{\sqrt{3^4\times5^2}}$

   $= \frac{3^5\times5^3}{3^2\times5}$

   $= 3^3\times5^2 \ (= 675)$

Surds Answers

1. (i) $\sqrt{8}=\sqrt{4\times 2}=\sqrt{4}\times\sqrt{2}=2\sqrt{2}$

   (ii) $\sqrt{50}=\sqrt{25\times 2}=\sqrt{25}\times\sqrt{2}=5\sqrt{2}$

   (iii) $\sqrt{48}=\sqrt{16\times 3}=\sqrt{16}\times\sqrt{3}=4\sqrt{3}$

   (iv) $\sqrt{216}=\sqrt{36\times 6}=\sqrt{36}\times\sqrt{6}=6\sqrt{6}$

2. (i) $\left(1+\sqrt{2}\right)+\left(3-2\sqrt{2}\right)=1+3+\sqrt{2}-2\sqrt{2}$
   $=4-\sqrt{2}$

   (ii) $\left(5\sqrt{2}-2\sqrt{3}\right)-\left(\sqrt{2}+3\sqrt{3}\right)=5\sqrt{2}-2\sqrt{3}-\sqrt{2}-3\sqrt{3}$
   $=4\sqrt{2}-5\sqrt{3}$

   (iii) $2\left(\sqrt{5}-3\sqrt{3}\right)+3\left(2\sqrt{5}+\sqrt{3}\right)=2\sqrt{5}-6\sqrt{3}+6\sqrt{5}+3\sqrt{3}$
   $=8\sqrt{5}-3\sqrt{3}$

3. (i) $\left(1+\sqrt{2}\right)\left(3-\sqrt{2}\right)=3-\sqrt{2}+3\sqrt{2}-2$
   $=1+2\sqrt{2}$

   (ii) $\left(2-\sqrt{3}\right)\left(3+2\sqrt{3}\right)=6+4\sqrt{3}-3\sqrt{3}-2\times 3$
   $=\sqrt{3}$

4. (i) $\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{3}=\sqrt{3}$

   (ii) $\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{5}}{5}$

   (iii) $\frac{1+\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\left(1+\sqrt{2}\right)\sqrt{2}}{2}=\frac{\sqrt{2}+2}{2}$

   (iv) $\frac{1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{\sqrt{3}-1}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\frac{\sqrt{3}-1}{3-1}=\frac{\sqrt{3}-1}{2}$

   (v) $\frac{\sqrt{2}}{2-\sqrt{2}}\times\frac{2+\sqrt{2}}{2+\sqrt{2}}=\frac{\sqrt{2}\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)}=\frac{2\sqrt{2}+2}{4-2}=\frac{2\sqrt{2}+2}{2}=\sqrt{2}+1$

Simultaneous Equations Answers

1. (i)

$$\begin{aligned} 2x + 5y &= 11 \quad (1) \\ 2x - y &= 5 \quad (2) \end{aligned}$$

Subtracting: $6y = 6$

$y = 1$

Substituting into (1): $2x + 5 \times 1 = 11$

$2x = 6$

$x = 3$

The solution is $x = 3$, $y = 1$. Check: $2x + 5y = 2 \times 3 + 5 \times 1 = 11$

$2x - y = 2 \times 3 - 1 = 5$

(ii) $x + 2y = 6$ (1)×4 $\quad 4x + 8y = 24$

$4x + 3y = 4$ (2) $\quad 4x + 3y = 4$

Subtracting: $5y = 20$

$y = 4$

Substituting into (1): $x + 2 \times 4 = 6$

$x = -2$

The solution is $x = -2$, $y = 4$. Check: $x + 2y = -2 + 8 = 6$

$4x + 3y = -8 + 12 = 4$

(iii) $3a - 2b = 4$ (1)×2 $\quad 6a - 4b = 8$

$5a + 4b = 3$ (2) $\quad 5a + 4b = 3$

Adding: $11a = 11$

$a = 1$

Substituting into (1): $3 \times 1 - 2b = 4$

$-2b = 1$

$b = -\frac{1}{2}$

The solution is $a = 1$, $b = -\frac{1}{2}$. Check: $3a - 2b = 3 + 1 = 4$

$5a + 4b = 5 - 2 = 3$

(iv) $2p - 5q = 5$ (1)×3 $\quad 6p - 15q = 15$

$3p - 2q = -9$ (2)×2 $\quad 6p - 4q = -18$

Subtracting: $-11q = 33$

$q = -3$

Substituting into (1): $2p - 5 \times -3 = 5$

$$2p = -10$$
$$p = -5$$

The solution is $p = -5$, $q = -3$. Check: $2p - 5q = -10 + 15 = 5$

$$3p - 2q = -15 + 6 = -9$$

2. (i)
$$x - y = -1 \quad (A)$$
$$3x + 2y = 7 \quad (B)$$
$$(A) \Rightarrow 2x - 2y = -2 \quad (C)$$
$$(C) + (B) \Rightarrow 5x = 5$$
$$\Rightarrow x = 1,\ y = 2$$

(ii)
$$2x + y = 0 \quad (A)$$
$$x - 3y = 7 \quad (B)$$
$$(B) \Rightarrow 2x - 6y = 14 \quad (C)$$
$$(A) - (C) \Rightarrow 7y = -14$$
$$\Rightarrow y = -2,\ x = 1$$

(iii)
$$y - 5x = -8 \quad (A)$$
$$x + 3y = 0 \quad (B)$$
$$(B) \Rightarrow 5x + 15y = 0 \quad (C)$$
$$(A) + (C) \Rightarrow 16y = -8$$
$$\Rightarrow y = -\tfrac{1}{2},\ x = \tfrac{3}{2}$$

(iv)
$$x = 2y - 1 \quad (A)$$
$$-x + 3y = -1 \quad (B)$$
$$(A) + (B) \Rightarrow 3y = 2y - 2 \quad (C)$$
$$\Rightarrow y = -2,\ x = -5$$

The Quadratic Formula Answers

1. (i) $3x^2+2x+5=0 \Rightarrow$ discriminant $=b^2-4ac$

$$=4-4.3.5<0$$

so no solutions

(ii) $2x^2-3x-2=0 \Rightarrow$ discriminant $=b^2-4ac$

$$=9-4(2)(-2)>0$$

so two solutions

(iii) $5x^2-6=0 \Rightarrow$ discriminant $=b^2-4ac$

$$=0-4.5(-6)>0$$

so two solutions

(iv) $4x^2-8x+4=0 \Rightarrow$ discriminant $=b^2-4ac$

$$=64-4(4)(4)=0$$

so two (equal) solutions

2. (i) $a=1, b=4, c=1$

$$x=\frac{-4\pm\sqrt{4^2-4\times1\times1}}{2\times1}=\frac{-4\pm\sqrt{12}}{2}=\frac{-4\pm2\sqrt{3}}{2}=-2\pm\sqrt{3}$$

(ii) $a=1, b=-3, c=-1$

$$x=\frac{3\pm\sqrt{3^2-4\times1\times-1}}{2\times1}=\frac{3\pm\sqrt{13}}{2}$$

(iii) $a=2, b=2, c=-3$

$$x=\frac{-2\pm\sqrt{2^2-4\times2\times-3}}{2\times2}=\frac{-2\pm\sqrt{28}}{4}=\frac{-2\pm2\sqrt{7}}{4}=\frac{-1\pm\sqrt{7}}{2}$$

Inequalities Answers

1. (i)
$$2x+3<10$$
$$2x<7$$
$$x<\tfrac{7}{2}$$

(ii)
$$5x+3\geq 2x-9$$
$$3x+3\geq -9$$
$$3x\geq -12$$
$$x\geq -4$$

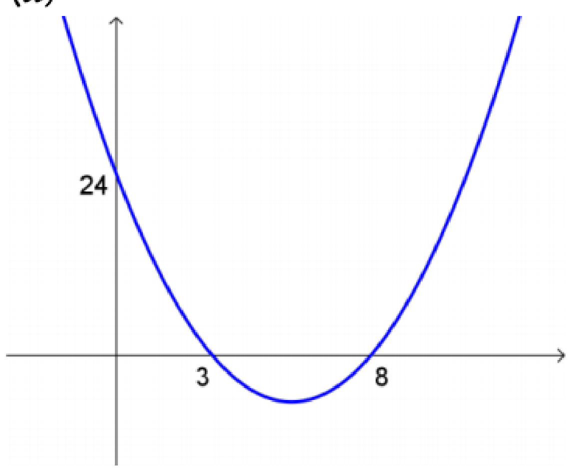
(iii)
$$3x-1>7-x$$
$$4x-1>7$$
$$4x>8$$
$$x>2$$

(iv)
$$4x+1\leq 6x-7$$
$$1\leq 2x-7$$
$$8\leq 2x$$
$$4\leq x$$
$$x\geq 4$$

(v)
$$5x+2>-7$$
$$5x>-9$$
$$x>-\tfrac{9}{5}$$

2. (i) $x^2-11x+24=(x-8)(x-3)$

(ii)

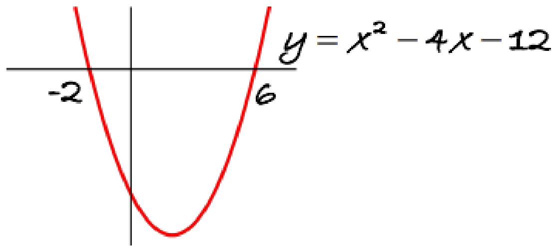


(iii) From the graph, $x^2-11x+24\geq 0 \Rightarrow x\leq 3$ or $x\geq 8$.

3. (i) $x^2 - 4x - 12 \le 0$

$(x-6)(x+2) \le 0$

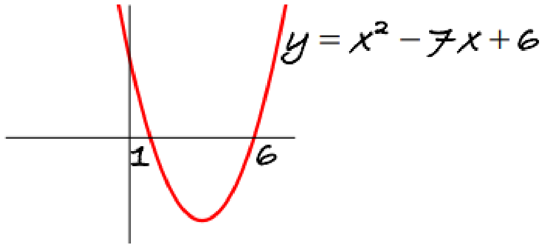
From graph, $-2 \le x \le 6$



(ii) $x^2 - 7x + 6 > 0$

$(x-1)(x-6) > 0$

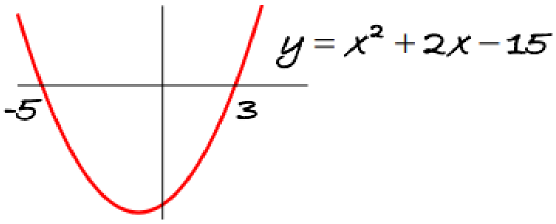
From graph, $x < 1$ or $x > 6$



(iii) $x^2 + 2x - 15 \ge 0$

$(x+5)(x-3) \ge 0$

From graph, $x \le -5$ or $x \ge 3$



4. (i) $x > 2$

Since the line is dotted, $x = 2$ is not included in the region

(ii) $y \ge 1$

(iii) $y > x$

(iv) $y \le x + 1$

Algebraic Fractions Answers

1. (i) $\dfrac{x^2 - 5x}{x-5} = \dfrac{x(\cancel{x-5})}{\cancel{x-5}} = x$

(ii) $\dfrac{x^2 + 6x - 16}{(x-2)(x-8)} = \dfrac{(x+8)(\cancel{x-2})}{(\cancel{x-2})(x-8)} = \dfrac{x+8}{x-8}$

(iii) $\dfrac{x^2 - a^2}{(x-a)^2} = \dfrac{(x+a)(\cancel{x-a})}{(x-a)^{\cancel{2}}} = \dfrac{x+a}{x-a}$

(iv) $\dfrac{v-3}{6-2v} = \dfrac{v-3}{2(3-v)} = \dfrac{-(\cancel{3-v})}{2(\cancel{3-v})} = -\dfrac{1}{2}$

2. (i) $\dfrac{x^2 + 3x - 4}{8} \times \dfrac{2}{3x-3} = \dfrac{(x+4)(\cancel{x-1})}{\cancel{8}4} \times \dfrac{\cancel{2}}{3(\cancel{x-1})} = \dfrac{x+4}{12}$

(ii) $\dfrac{3}{x+2}\times\dfrac{x^2-4x-12}{x^2-2x-24}=\dfrac{3}{\cancel{x+2}}\times\dfrac{(\cancel{x-6})(\cancel{x+2})}{(\cancel{x-6})(x+4)}=\dfrac{3}{x+4}$

3. (i) $\dfrac{5c+15}{2}\div\dfrac{c^2-9}{4}=\dfrac{5c+15}{2}\times\dfrac{4}{c^2-9}$

$=\dfrac{5(\cancel{c+3})}{2}\times\dfrac{4}{(\cancel{c+3})(c-3)}$

$=\dfrac{10}{c-3}$

(ii) $\dfrac{x^2-x}{2x+1}\div\dfrac{x^2+2x-3}{2x^2-3x-2}=\dfrac{x^2-x}{2x+1}\times\dfrac{2x^2-3x-2}{x^2+2x-3}$

$=\dfrac{x(\cancel{x-1})}{\cancel{2x+1}}\times\dfrac{(\cancel{2x+1})(x-2)}{(x+3)(\cancel{x-1})}$

$=\dfrac{x(x-2)}{x+3}$

4. (i) $\dfrac{x}{2}+\dfrac{3x}{5}=\dfrac{5x}{10}+\dfrac{6x}{10}=\dfrac{11x}{10}$

(ii) $\dfrac{2}{3x}-\dfrac{1}{4x}=\dfrac{8}{12x}-\dfrac{3}{12x}=\dfrac{5}{12x}$

5. (i) $\dfrac{2}{x+3}+\dfrac{1}{x-5}=\dfrac{2(x-5)}{(x+3)(x-5)}+\dfrac{x+3}{(x+3)(x-5)}$

$=\dfrac{2x-10+x+3}{(x+3)(x-5)}$

$=\dfrac{3x-7}{(x+3)(x-5)}$

(ii) $\dfrac{5}{2x-3}-\dfrac{2}{4x+1}=\dfrac{5(4x+1)}{(2x-3)(4x+1)}-\dfrac{2(2x-3)}{(2x-3)(4x+1)}$

$=\dfrac{20x+5-4x+6}{(2x-3)(4x+1)}$

$=\dfrac{16x+11}{(2x-3)(4x+1)}$

6. (i) $$\frac{1}{2(x+5)}-\frac{1}{4(x+1)}=\frac{2(x+1)}{4(x+5)(x+1)}-\frac{x+5}{4(x+5)(x+1)}$$
$$=\frac{2x+2-x-5}{4(x+5)(x+1)}$$
$$=\frac{x-3}{4(x+5)(x+1)}$$

7. (i) Since a cubic is being divided by a linear expression, the quotient is quadratic and the remainder constant.

$$\frac{3x^3-x^2+2x-4}{x+2}=Ax^2+Bx+C+\frac{D}{x+2}$$
$$3x^3-x^2+2x-4=(Ax^2+Bx+C)(x+2)+D$$
$$=Ax^3+(2A+B)x^2+(2B+C)x+2C+D$$

Equating coefficients of x^3: $A=3$

Equating coefficients of x^2: $2A+B=-1 \Rightarrow B=-7$

Equating coefficients of x: $2B+C=2 \Rightarrow C=16$

Equating constant terms: $2C+D=-4 \Rightarrow D=-36$

$$\frac{3x^3-x^2+2x-4}{x+2}=3x^2-7x+16-\frac{36}{x+2}$$

(ii) Since a quartic is being divided by a quadratic, the quotient is quadratic and the remainder linear.

$$\frac{x^4+2x^2-x-1}{x^2+1}=Ax^2+Bx+C+\frac{Dx+E}{x^2+1}$$
$$x^4+2x^2-x-1=\left(Ax^2+Bx+C\right)\left(x^2+1\right)+Dx+E$$
$$=Ax^4+Bx^3+(A+C)x^2+(B+D)x+C+E$$

Equating coefficients of x^4: $A=1$

Equating coefficients of x^3: $B=0$

Equating coefficients of x^2: $A+C=2 \Rightarrow C=1$

Equating coefficients of x: $B+D=-1 \Rightarrow D=-1$

Equating constant terms: $C+E=-1 \Rightarrow E=-2$

$$\frac{x^4+2x^2-x-1}{x^2+1}=x^2+1+\frac{-x-2}{x^2+1}$$

(iii) Since a cubic is being divided by a quadratic, the quotient is linear and the remainder linear.

$$\frac{x^3}{x^2+2} = Ax + B + \frac{Cx+D}{x^2+2}$$

$$x^3 = (Ax+B)(x^2+2) + Cx + D$$

$$= Ax^3 + Bx^2 + (2A+C)x + 2B + D$$

Equating coefficients of x^3: $A = 1$

Equating coefficients of x^2: $B = 0$

Equating coefficients of x: $2A + C = 0 \Rightarrow C = -2$

Equating constant terms: $2B + D = 0 \Rightarrow D = 0$

$$\frac{x^3}{x^2+2} = x + \frac{-2x}{x^2+2}$$

(iv) Since a linear expression is being divided by a linear expression, the quotient is a constant and the remainder a constant.

$$\frac{6x-2}{2x+3} = A + \frac{B}{2x+3}$$

$$6x - 2 = A(2x+3) + B$$

$$= 2Ax + 3A + B$$

Comparing coefficients of x: $2A = 6 \Rightarrow A = 3$

Comparing constant terms: $3A + B = -2 \Rightarrow B = -11$

$$\frac{6x-2}{2x+3} = 3 - \frac{11}{2x+3}$$

(v) Since a quadratic expression is being divided by a quadratic expression, the quotient is a constant and the remainder linear.

$$\frac{3x^2+1}{x^2-2x-1} = A + \frac{Bx+C}{x^2-2x-1}$$

$$3x^2 + 1 = A(x^2-2x-1) + Bx + C$$

$$= Ax^2 + (B-2A)x - A + C$$

Comparing coefficients of x^2: $A = 3$

Comparing coefficients of x: $B - 2A = 0 \Rightarrow B = 6$

Comparing constant terms: $-A + C = 1 \Rightarrow C = 4$

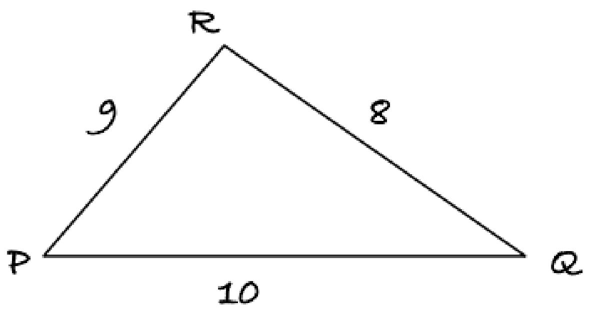
$$\frac{3x^2+1}{x^2-2x-1} = 3 + \frac{6x+4}{x^2-2x-1}$$

The Sine and Cosine Rules

1. Using the sine rule:
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin A}{16} = \frac{\sin 30^\circ}{10}$$
$$\sin A = \frac{16 \sin 30^\circ}{10} = 0.8$$
$$A = 53.1^\circ \text{ or } 126.9^\circ$$
$$C = 180^\circ - 30^\circ - A = 96.1^\circ \text{ or } 23.1^\circ$$

Using the sine rule:
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin C} = \frac{10}{\sin 30^\circ}$$
$$c = \frac{10 \sin C}{\sin 30^\circ} = 19.9 \text{ cm or } 7.9 \text{ cm}$$

2.

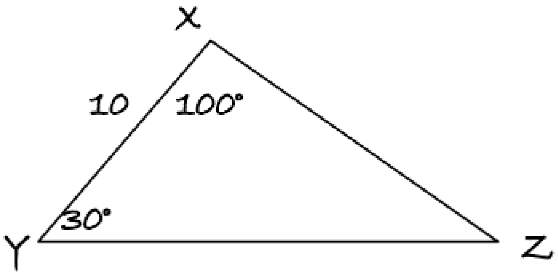


Using the cosine rule:
$$\cos P = \frac{q^2 + r^2 - p^2}{2qr} = \frac{9^2 + 10^2 - 8^2}{2 \times 9 \times 10}$$
$$P = 49.5^\circ$$

Using the cosine rule:
$$\cos Q = \frac{p^2 + r^2 - q^2}{2pr} = \frac{8^2 + 10^2 - 9^2}{2 \times 8 \times 10}$$
$$Q = 58.8^\circ$$

$$R = 180^\circ - 49.46^\circ - 58.75^\circ = 71.8^\circ$$

3.



Angle Z = 180° - 100° - 30° = 50°

Using the sine rule:
$$\frac{x}{\sin X} = \frac{z}{\sin z}$$
$$\frac{x}{\sin 100^\circ} = \frac{10}{\sin 50^\circ}$$
$$x = \frac{10 \sin 100^\circ}{\sin 50^\circ} = 12.86$$

$$\text{Area of triangle} = \tfrac{1}{2} xz \sin Y$$
$$= \tfrac{1}{2} \times 12.86 \times 10 \sin 30^\circ$$
$$= 32.1 \text{ cm}^2$$

4. Let $a = 6$ and $b = 7$

$\text{Area of triangle} = \tfrac{1}{2} ab \sin C$

$12 = \tfrac{1}{2} \times 6 \times 7 \sin C$

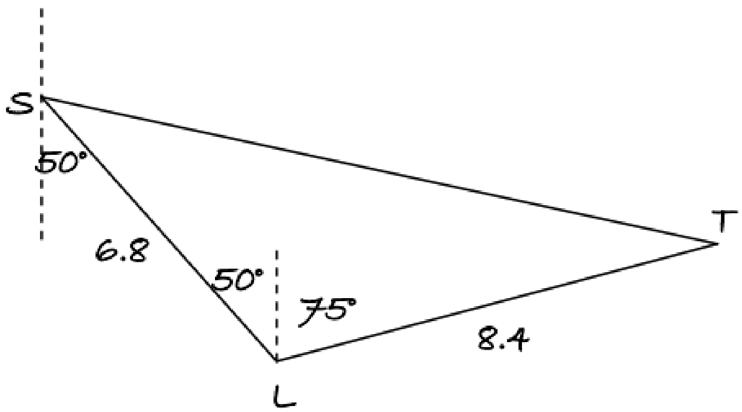
$C = 34.85^\circ$ or 145.15°

Using the cosine rule:
$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$= 6^2 + 7^2 - 2 \times 6 \times 7 \cos C$$
$$= 85 - 84 \cos C$$

If $C = 34.85^\circ$, $c = 4.01$ cm

If $C = 145.14^\circ$, $c = 12.41$ cm

5.



Using cosine rule: $ST^2 = 6.8^2 + 8.4^2 - 2 \times 6.8 \times 8.4 \cos 125°$

$ST = 13.5$ km

Using sine rule: $\dfrac{\sin S}{8.4} = \dfrac{\sin 125°}{13.5}$

$$\sin S = \frac{8.4 \sin 125°}{13.5}$$

$$S = 30.6°$$

Bearing of T from S = 180° - 50° - 30.6° = 099.4°

6. (i) $\dfrac{\sin 10}{8} = \dfrac{\sin \beta}{6}$

$$\Rightarrow \sin \beta = 0.13$$

$$\Rightarrow \beta \approx 7.48°$$

$$\Rightarrow \alpha = 180° - 10° - 7.48° \approx 162.52°$$

$$\Rightarrow a^2 = 6^2 + 8^2 - 2(6)(8)\cos 162.52° \approx 191.57$$

$$\Rightarrow a \approx 13.8$$

(ii) $b^2 = 6^2 + 7^2 - 2(6)(7)\cos 50° \approx 31.009$

$$\Rightarrow b \approx 5.57$$

$$\frac{\sin \beta}{7} = \frac{\sin 50°}{5.57} \Rightarrow \sin \beta \approx 0.933$$

$$\Rightarrow \beta \approx 74.3°$$

The Sine and Cosine Rules

1. (i) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1-y}{3-6} = \dfrac{1-y}{-3}$

Gradient of AB $= 2 \Rightarrow \dfrac{1-y}{-3} = 2$

$$\Rightarrow 1 - y = -6$$

$$y = 7$$

(ii) Distance AB is 5

$$\sqrt{(3-6)^2+(1-y)^2}=5$$
$$9+(1-y)^2=25$$
$$(1-y)^2=16$$
$$1-y=\pm 4$$
$$y=1-4 \text{ or } 1+4$$
$$y=-3 \text{ or } 5$$

(iii) If A, B and C are collinear, gradient of AB = gradient of AC.

Gradient of AC $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{1-(-2)}{3-12}=\dfrac{3}{-9}=-\dfrac{1}{3}$

From (i), gradient of AB $=\dfrac{1-y}{-3}$

$$\frac{1-y}{-3}=-\frac{1}{3}$$
$$1-y=1$$
$$y=0$$

2. (i) Gradient of $y=4x-1$ is 4

Gradient of parallel line = 4

Equation of line is $y-3=4(x-2)$

$$y-3=4x-8$$
$$y=4x-5$$

(ii) Gradient of $y=2x+7$ is 2

Gradient of perpendicular line is $-\frac{1}{2}$

Equation of line is $y-2=-\frac{1}{2}(x-1)$

$$2(y-2)=-(x-1)$$
$$2y-4=-x+1$$
$$2y+x=5$$

3. (i) Gradient of AB $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{6-2}{1-3}=\dfrac{4}{-2}=-2$

Equation of AB is $y-6=-2(x-1)$

$$y-6=-2x+2$$
$$y+2x=8$$

(ii) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-1-3}{8-(-2)} = \dfrac{-4}{10} = -\dfrac{2}{5}$

Equation of AB is $y-(-1) = -\frac{2}{5}(x-8)$

$$5(y+1) = -2(x-8)$$
$$5y+5 = -2x+16$$
$$5y+2x = 11$$

4. Gradient of EF $= \dfrac{3-(-1)}{1-2} = \dfrac{4}{-1} = -4$

Gradient of FG $= \dfrac{5-3}{3-1} = \dfrac{2}{2} = 1$

Gradient of GH $= \dfrac{1-5}{4-3} = \dfrac{-4}{1} = -4$

Gradient of EH $= \dfrac{1-(-1)}{4-2} = \dfrac{2}{2} = 1$

EF is parallel to GH and FG is parallel to EH so EFGH is a parallelogram.

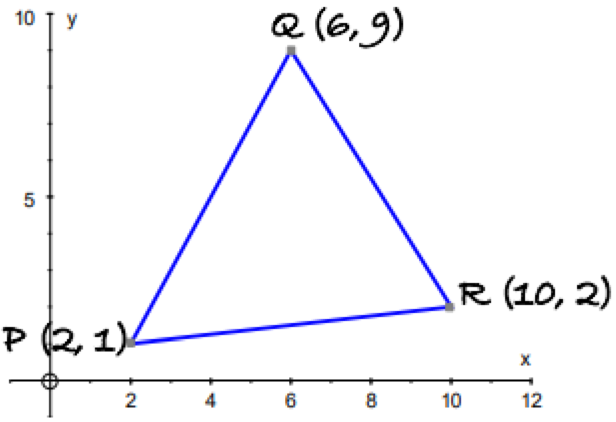
If EFGH were a rhombus, all the sides would be equal.

$EF^2 = (2-1)^2 + (-1-3)^2 = 1^2 + (-4)^2 = 17$

$FG^2 = (1-3)^2 + (3-5)^2 = (-2)^2 + (-2)^2 = 8$

The lengths of EF and FG are not equal, so EFGH is not a rhombus.

5. (i)



(ii) $PQ = \sqrt{(6-2)^2 + (9-1)^2} = \sqrt{16+64} = \sqrt{80}$

$PR = \sqrt{(10-2)^2 + (2-1)^2} = \sqrt{64+1} = \sqrt{65}$

$QR = \sqrt{(10-6)^2 + (2-9)^2} = \sqrt{16+49} = \sqrt{65}$

Since PR and QR are the same length, the triangle is isosceles.

(iii) Take the base of the triangle as PQ
Let M be the midpoint of PQ

$$M = \left(\frac{2+6}{2}, \frac{1+9}{2}\right) = (4,5)$$

Height of triangle is $MR = \sqrt{(10-4)^2 + (2-5)^2} = \sqrt{36+9} = \sqrt{45}$

$$\begin{aligned}\text{Area of triangle} &= \tfrac{1}{2} \times PQ \times MR \\ &= \tfrac{1}{2}\sqrt{80}\sqrt{45} \\ &= \tfrac{1}{2}\sqrt{16\times 5}\sqrt{9\times 5} \\ &= \tfrac{1}{2}\times 4\sqrt{5}\times 3\sqrt{5} \\ &= 6\times 5 \\ &= 30\end{aligned}$$

Circles

1. (i) $(x-0)^2 + (y-0)^2 = 6^2$
$x^2 + y^2 = 36$

(ii) $(x-3)^2 + (y-1)^2 = 5^2$
$x^2 - 6x + 9 + y^2 - 2y + 1 = 25$
$x^2 + y^2 - 6x - 2y = 15$

2. (i) $x^2 + y^2 = 100 = 10^2$
Centre = (0, 0), radius = 10.

(ii) $(x-2)^2 + (y-7)^2 = 16 = 4^2$
Centre = (2, 7), radius = 4

(iii) $(x+3)^2 + (y-4)^2 = 4 = 2^2$
Centre = (-3, 4), radius = 2

(iv) $(x+4)^2 + (y+5)^2 = 20$
Centre = (-4, -5), radius $= \sqrt{20}$

3. (i) $x^2 + y^2 + 4x - 5 = 0$
$x^2 + 4x + y^2 - 5 = 0$
$(x+2)^2 - 4 + y^2 - 5 = 0$
$(x+2)^2 + y^2 = 9$
Centre = (-2, 0), radius = 3.

(ii) $x^2 + y^2 - 6x + 10y + 20 = 0$
$x^2 - 6x + y^2 + 10y + 20 = 0$
$(x-3)^2 - 9 + (y+5)^2 - 25 + 20 = 0$
$(x-3)^2 + (y+5)^2 = 14$
Centre is (3, -5) and radius $= \sqrt{14}$

4. Radius of circle $= \sqrt{(6-4)^2+(3-(-2))^2} = \sqrt{4+25} = \sqrt{29}$

Equation of circle is $(x-4)^2+(y+2)^2 = 29$

$$x^2-8x+16+y^2+4y+4=29$$
$$x^2+y^2-8x+4y=9$$

5. Centre of circle C is the midpoint of AB.

$$C=\left(\frac{2+6}{2},\frac{0+4}{2}\right)=(4,2)$$

Radius of circle is distance AC $= \sqrt{(2-4)^2+(0-2)^2} = \sqrt{4+4} = \sqrt{8}$

Equation of circle is $(x-4)^2+(y-2)^2 = 8$

$$x^2-8x+16+y^2-4y+4=8$$
$$x^2+y^2-8x-4y+12=0$$

Coordinate Geometry

1. (i) $5y+4x=3$.

$$5y=-4x+3$$
$$y=-\tfrac{4}{5}x+\tfrac{3}{5}$$

Gradient of line $= -\tfrac{4}{5}$

[1]

(ii) l_2 is parallel to l_1, so it has gradient $-\tfrac{4}{5}$.

Equation of line is $y-(-2)=-\tfrac{4}{5}(x-1)$

$$5(y+2)=-4(x-1)$$
$$5y+10=-4x+4$$
$$5y+4x+6=0$$

[3]

2. The curve is a circle, centre O and radius 2.

[2]

3. Gradient of AB $= \dfrac{y_1-y_2}{x_1-x_2} = \dfrac{-3-7}{-1-5} = \dfrac{-10}{-6} = \dfrac{5}{3}$

Gradient of line perpendicular to AB $= -\dfrac{3}{5}$.

The line passes through the midpoint of AB $= \left(\dfrac{-1+5}{2},\dfrac{-3+7}{2}\right)=(2,2)$

Equation of line is $y-2=-\tfrac{3}{5}(x-2)$

$$5(y-2)=-3(x-2)$$
$$5y-10=-3x+6$$
$$5y+3x=16$$

[4]

4. Substituting $y = 3x - 10$ into $x^2 + y^2 = 10$

gives $x^2 + (3x - 10)^2 = 10$

$$x^2 + 9x^2 - 60x + 100 = 10$$
$$10x^2 - 60x + 90 = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x - 3)^2 = 0$$

Since the equation has a repeated root, the line meets the circle just once, and so the line is a tangent to the circle.

[4]

5. (i) Substituting $y = 2x - 3$ into $3y + 4x = 8$:

$$3(2x - 3) + 4x = 8$$
$$6x - 9 + 4x = 8$$
$$10x = 17$$
$$x = 1.7$$

When $x = 1.7$, $y = 2 \times 1.7 - 3 = 3.4 - 3 = 0.4$

The coordinates of R are (1.7, 0.4)

[4]

(ii) P is the point on $y = 2x - 3$ where $y = 0$, so P is (1.5, 0)

Q is the point on $3y + 4x = 8$ where $y = 0$, so Q is (2, 0).

$$\begin{aligned} \text{Area} &= \tfrac{1}{2} \times \text{base} \times \text{height} \\ &= \tfrac{1}{2} \times 0.5 \times 0.4 \\ &= 0.1 \end{aligned}$$

Coordinate Geometry

1. (i) $$\begin{aligned} f(x) + g(x) &= (x^3 + 2x^2 - 5x + 4) + (x^3 - 3x^2 + 1) \\ &= x^3 + 2x^2 - 5x + 4 + x^3 - 3x^2 + 1 \\ &= x^3 + x^3 + 2x^2 - 3x^2 - 5x + 4 + 1 \\ &= 2x^3 - x^2 - 5x + 5 \end{aligned}$$

(ii) $$\begin{aligned} f(x) - g(x) &= (x^3 + 2x^2 - 5x + 4) - (x^3 - 3x^2 + 1) \\ &= x^3 + 2x^2 - 5x + 4 - x^3 + 3x^2 - 1 \\ &= x^3 - x^3 + 2x^2 + 3x^2 - 5x + 4 - 1 \\ &= 5x^2 - 5x + 3 \end{aligned}$$

2. (i) $q(x)-p(x)=(x^3-2x^2+1)-(2x^3-5x^2+3x-2)$

$$=x^3-2x^2+1-2x^3+5x^2-3x+2$$
$$=x^3-2x^3-2x^2+5x^2-3x+1+2$$
$$=-x^3+3x^2-3x+3$$

(ii) $2p(x)+3q(x)=2(2x^3-5x^2+3x-2)+3(x^3-2x^2+1)$

$$=4x^3-10x^2+6x-4+3x^3-6x^2+3$$
$$=4x^3+3x^3-10x^2-6x^2+6x-4+3$$
$$=7x^3-16x^2+6x-1$$

3. (i) $(x-2)(2x^2-3x+1)=x(2x^2-3x+1)-2(2x^2-3x+1)$

$$=2x^3-3x^2+x-4x^2+6x-2$$
$$=2x^3-7x^2+7x-2$$

(ii) $(3x-2)(x^3-2x+4)=3x(x^3-2x+4)-2(x^3-2x+4)$

$$=3x^4-6x^2+12x-2x^3+4x-8$$
$$=3x^4-2x^3-6x^2+16x-8$$

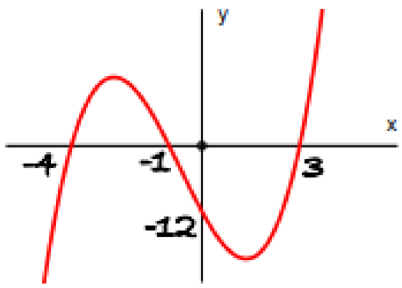
4. (i) $y=(x+1)(x-3)(x+4)$

This is a cubic graph which cuts the x-axis at (-1, 0), (3, 0) and (-4, 0).

When $x=0$, $y=1\times-3\times4=-12$

When x is large and positive, y is positive.

When x is large and negative, y is negative.



(ii) $y=(x+2)^2(2x-1)$

This is a cubic graph which touches the x-axis at (-2, 0) and cuts the x-axis at $(\frac{1}{2},0)$.

When $x=0$, $y=2^2\times-1=-4$

When x is large and positive, y is positive.

When x is large and negative, y is negative.